Comparing the Expressiveness of Argumentation Semantics[1]

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Abstract. In this work we complement recent investigations of the intertranslatability of argumentation semantics. Our focus is on the expressiveness of argumentation semantics and thus we expand the area of interest beyond efficiently computable translations. To this end we provide new translations between semantics as well as new translational impossibility results. This allows us to draw a hierarchy for the expressiveness of argumentation semantics.

Keywords. abstract argumentation, argumentation semantics, intertranslatability

1. Introduction

We investigate the intertranslatability of abstract argumentation semantics, i.e. whether it is possible to modify an arbitrary argumentation framework such that the σ-extensions of the original framework are in a certain correspondence with the σ'-extensions of the modified framework (σ, σ' being argumentation semantics).

In his seminal paper Dung [6] already proposed a broad range of argumentation semantics which since then was enhanced by the community (see e.g. [1] for an extensive overview). Inevitably when dealing with different semantics the question arises what kind of characteristics the difference affects. To this end basic properties [2] as well as the computational behaviour [7] of semantics have been extensively studied in the literature. Studies on intertranslatability of semantics complement the perception of argumentation semantics by relating semantics wrt. their expressiveness. With being able to translate one semantics into another immediately we are also able to interlock respective extensions and thus provide some sort of directed logical equivalence. On the other hand if one semantics cannot be translated into another we conclude that the first semantics provides certain expressiveness that cannot be simulated by the other semantics. Such investigations come into play in so called meta-level argumentation (e.g. [9]), where one wants to express certain semantics within another, for instance for the purpose of merging two frameworks with different corresponding semantics.

Intertranslatability results also affect more complex argumentation procedures. In this context we think about frameworks being instantiated from some (logical) knowledge-base, where the aim is to retrieve extensions satisfying specific rationality

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postulates wrt. the original knowledge-base (see e.g. [3]). One is thus only interested in semantics ensuring such extensions. Given some translation from one semantics σ to another semantics σ' and an instantiation such that the conclusions provided by σ satisfy the desired postulates one can build a similar instantiation for σ' by concatenating the original instantiation and the translation.

Prior investigations for translating argumentation semantics are to be found in [8], where complexity theoretic reductions are transferred to the area of abstract argumentation. This work is motivated mainly by computational issues, e.g. generalising existing solvers for application to various semantics, and thus focuses on translation functions that are efficiently computable. In contrast, our work goes beyond efficient translations and also considers translations making use of any desired computational resources. This is due to the fact that we are interested in the expressiveness of argumentation semantics.

In this work we will consider the semantics proposed in Dung's seminal paper [6] as well as stage [10] and semi-stable [4,10] semantics. We study two kinds of translations, exact translations, where the extensions of the original framework and the modified framework are identical, and faithful translations, where the extensions of the original framework and the translated framework agree on the original arguments but the extensions of the modified framework may contain additional arguments introduced by the translation. Depending on the concrete application additional arguments in an extension might be appropriate or not. As we will see later the different notions of intertranslatability lead to different hierarchies of expressiveness.

This work contributes in the following ways. First, we complement results from [8] by providing additional efficient translations for conflict-free and naive semantics, as well as an explicit translation for complete to preferred semantics which also improves a translation from [8]. Second, we present novel (inefficient) translations in cases where no efficient translation is possible. Finally, we provide negative results stating that translations between certain semantics are impossible independently of computational constraints and thus strengthen existing results.

The remainder of the paper is organised as follows: In Section 2 we present necessary background information for abstract argumentation and necessary notions for translations. On the positive side, in Section 3, we present translations between argumentation semantics. In Section 4 we present negative results showing that certain translations are not possible. Finally, in Section 5, we conclude with a summary of the results.

2. Background

2.1. Abstract Argumentation

In this section we introduce (abstract) argumentation frameworks [6] and recall the semantics we study in this paper (see also [1], for an overview).

Definition 1. *An* argumentation framework (AF) *is a pair $F = (A, R)$ where A is a finite and non-empty* [2] *set of arguments and $R \subseteq A \times A$ represents the attack relation. For a given AF $F = (A, R)$ we use A_F to denote the set A of its arguments and R_F to denote its attack relation R. The pair $(a, b) \in R$ means that a attacks b.*

[2] For technical reasons we only consider AFs with $A \neq \emptyset$.

We sometimes use the notation $a \rightarrowtail^R b$ instead of $(a,b) \in R$. For $S \subseteq A$ and $a \in A$, we also write $S \rightarrowtail^R a$ (resp. $a \rightarrowtail^R S$) in case there exists an argument $b \in S$, such that $b \rightarrowtail^R a$ (resp. $a \rightarrowtail^R b$). In case no ambiguity arises, we use $\rightarrowtail$ instead of $\rightarrowtail^R$.

An AF can naturally be represented as a directed graph. Semantics for argumentation frameworks are given via a function σ which assigns to each AF $F = (A,R)$ a set $\sigma(F) \subseteq 2^A$ of extensions. In place of σ we will consider the functions *cf*, *naive*, *stb*, *adm*, *prf*, *com*, *grd*, *stage*, and *sem* which stand for conflict-free, naive, stable, admissible, preferred, complete, grounded, stage, and respectively, semi-stable semantics. Before giving the actual definitions for these semantics, we require a few more formal concepts.

Definition 2. *Given an AF $F = (A,R)$, an argument $a \in A$ is* defended *(in F) by a set $S \subseteq A$ if for each $b \in A$, such that $b \rightarrowtail a$, also $S \rightarrowtail b$ holds. Moreover, for a set $S \subseteq A$, we define the* range *of S, denoted as S^+_R, as the set $S \cup \{b \mid S \rightarrowtail b\}$.*

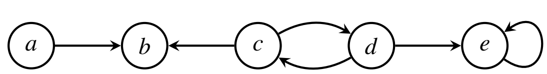
We continue with the definitions of the considered semantics. Observe that their common feature is the concept of conflict-freeness, i.e. arguments in an extension are not allowed to attack each other.

Definition 3. *Let $F = (A,R)$ be an AF. A set $S \subseteq A$ is* conflict-free (in F)*, denoted as $S \in cf(F)$, if there are no $a,b \in S$, such that $(a,b) \in R$. For $S \in cf(F)$, it holds that*

- *$S \in naive(F)$, if there is no $T \in cf(F)$ with $T \supset S$;*
- *$S \in stb(F)$, if for each $a \in A \setminus S$, $S \rightarrowtail a$, i.e. $S^+_R = A$;*
- *$S \in adm(F)$, if each $a \in S$ is defended by S;*
- *$S \in prf(F)$, if $S \in adm(F)$ and there is no $T \in adm(F)$ with $T \supset S$;*
- *$S \in com(F)$, if $S \in adm(F)$ and for each $a \in A$ that is defended by S, $a \in S$;*
- *$S \in grd(F)$, if $S \in com(F)$ and there is no $T \in com(F)$ with $T \subset S$;*
- *$S \in stage(F)$, if there is no conflict-free set T in F, such that $T^+_R \supset S^+_R$;*
- *$S \in sem(F)$, if $S \in adm(F)$ and there is no $T \in adm(F)$ with $T^+_R \supset S^+_R$.*

We recall that for any AF F, $stb(F) \subseteq sem(F) \subseteq prf(F) \subseteq com(F) \subseteq adm(F)$ holds, and that for each of the considered semantics σ except stable semantics, $\sigma(F) \neq \emptyset$ holds. The grounded semantics always yields exactly one extension. Moreover if an AF has at least one stable extension then stable, semi-stable, and stage semantics coincide.

Example 1. Consider the AF $F = (A,R)$, with $A = \{a,b,c,d,e\}$ and $R = \{(a,b), (c,b), (c,d), (d,c), (d,e), (e,e)\}$. The graph representation of F is given as follows.



We have $stb(F) = stage(F) = sem(F) = \{\{a,d\}\}$. Further we have $adm(F) = \{\{\},\{a\},\{c\},\{d\},\{a,c\},\{a,d\}\}$, thus $prf(F) = \{\{a,c\},\{a,d\}\}$ and $cf(F) = adm(F) \cup \{\{b\},\{b,d\}\}$, thus $naive(F) = prf(F) \cup \{\{b,d\}\}$. Finally the complete extensions of F are $\{a\}$, $\{a,c\}$ and $\{a,d\}$, with $\{a\}$ being the grounded extension of F. $\Diamond$

2.2. Translations

In what follows, we understand as a translation *Tr* a function which maps AFs to AFs. In particular, we seek translations, such that for given semantics σ, σ', the extensions

$\sigma(F)$ are in a certain relation to extensions $\sigma'(Tr(F))$ for each AF F. Following [8], we introduce a few additional properties which seem desirable for such translations. To this end, we define, for AFs $F = (A,R)$, $F' = (A',R')$, the union of AFs as $F \cup F' = (A \cup A', R \cup R')$, and inclusion as $F \subseteq F'$ iff jointly $A \subseteq A'$ and $R \subseteq R'$.

Definition 4. *A translation Tr is called*

- efficient *if for every AF F, the AF $Tr(F)$ can be computed using logarithmic space wrt. to $|F|$;*
- covering *if for every AF F, $F \subseteq Tr(F)$;*
- embedding *if for every AF F, $A_F \subseteq A_{Tr(F)}$ and $R_F = R_{Tr(F)} \cap (A_F \times A_F)$;*
- monotone *if for any AFs F, F', $F \subseteq F'$ implies $Tr(F) \subseteq Tr(F')$;*
- modular *if for any AFs F, F', $Tr(F) \cup Tr(F') = Tr(F \cup F')$.*

It is easy to see that each embedding translation is also covering and that each modular translation is also monotone. For a deeper discussion of these properties the interested reader is referred to [8]. Next, in accordance with [8], we give different notions of how extensions of the original AF and the modified AF correspond to each other.

Definition 5. *Let σ, σ' be semantics. We call a translation Tr*

- exact *for $\sigma \Rightarrow \sigma'$ if for every AF F, $\sigma(F) = \sigma'(Tr(F))$;*
- faithful *for $\sigma \Rightarrow \sigma'$ if for every AF F, $\sigma(F) = \{E \cap A_F \mid E \in \sigma'(Tr(F))\}$ and $|\sigma(F)| = |\sigma'(Tr(F))|$.*
- weakly exact *for $\sigma \Rightarrow \sigma'$ if there exists a collection $\mathscr{S}$ of sets of arguments, such that for any AF F, $\sigma(F) = \sigma'(Tr(F)) \setminus \mathscr{S}$;*
- weakly *faithful for $\sigma \Rightarrow \sigma'$ if there exists a collection $\mathscr{S}$ of sets of arguments, such that for any AF F, $\sigma(F) = \{E \cap A_F \mid E \in \sigma'(Tr(F)) \setminus \mathscr{S}\}$ and $|\sigma(F)| = |\sigma'(Tr(F)) \setminus \mathscr{S}|$.*

The notion of "weakly" exact / faithful is because we have to face that, for some semantics some AFs do not possess an extension, while other semantics always yield at least one extension and further that there are semantics where the empty set is always an extension. We sometimes refer to the elements from $\mathscr{S}$ as remainder sets. Note that $\mathscr{S}$ depends only on the translation, but not on the input AF. Thus, by definition, each $S \in \mathscr{S}$ only contains arguments which never occur in AFs subject to translation. In other words, we reserve certain arguments for introduction in weak translations.

All the properties from Definition 4 as well as the properties of being exact, weakly exact and faithful are transitive, i.e. for two transformations satisfying one of these properties, also the concatenation satisfies the respective property. However, transitivity is not guaranteed for being weakly faithful. Next we present a new relation between the properties efficiency and modularity.

Proposition 1. *Any modular translation is already efficient.*

Proof. We look at an arbitrary AF $F = (A,R)$ and investigate some modular translation Tr. By the definition of modularity we have $Tr(F) = \bigcup_{G \subseteq F, |G| \leq 2} Tr(G)$. Notice that naming of arguments is irrelevant and thus a modular translation is fully determined by its translations of all AFs over arguments a, b. This gives us a finite number of graph patterns of bounded size.

Table 1. Results for (weakly) faithful / exact translations (state of the art).

	cf	*naive*	*grd*	*adm*	*stb*	*com*	*prf*	*sem*	*stage*
cf	✓								
naive		✓							
grd			✓	✓/ ?	✓/ ?	✓/ ?	✓/ ?	✓/ ?	✓/ ?
adm			–	✓	✓/ -	✓	✓/ -	✓/ -	✓/ -
stb			–	✓	✓	✓	✓	✓	✓
com			–	✓/ -	✓/ -	✓	✓/ -	✓/ -	✓/ -
prf			–				✓	✓	? / -
sem			–					✓	? / -
stage			–					✓	✓

To translate an AF F, for each of these graph patterns we have to identify isomorphic subgraphs of F and apply Tr to these subgraphs. As the patterns are fixed searching for isomorphic subgraphs can be done in logarithmic space and as also the translation of the patterns is fixed also translations of isomorphic subgraphs are in logarithmic space. $\square$

In Table 1 we summarize existing results on the intertranslatability of semantics taken from [8] (recall that in contrast to [8] we do not require translations to be efficient). An entry in row σ and column σ' of Table 1 is to be read as follows: "✓" there is a (weakly) exact translation for $\sigma \Rightarrow \sigma'$; "✓/-" there is a (weakly) faithful translation, but there does not exist a (weakly) exact translation, for $\sigma \Rightarrow \sigma'$; "✓/ ?" there is a (weakly) faithful translation, but it is not known whether there exist a (weakly) exact translation, for $\sigma \Rightarrow \sigma'$; " ? / -" there does not exist a (weakly) exact translation, but it is not known whether there exist a (weakly) faithful translation, for $\sigma \Rightarrow \sigma'$; "–" there is no (weakly) faithful translation for $\sigma \Rightarrow \sigma'$.

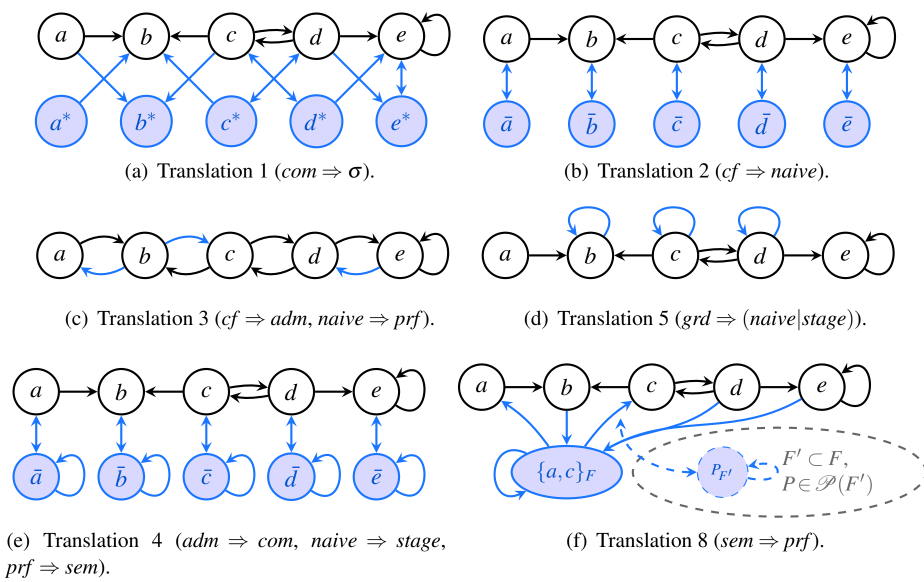
3. Translations between Semantics

In this Section we present translations between the semantics of interest. In the following, given a set of arguments A, we write A^* (resp. $\bar{A}$) to denote the set of new arguments $\{a^* \mid a \in A\}$ (resp. $\{\bar{a} \mid a \in A\}$). We start with a translation mapping complete semantics to stage, stable, semi-stable and preferred semantics. While the intertranslatability of these semantics was already shown in [8], the given translation improves existing results in two directions. First, it gives an explicit translation for $com \Rightarrow prf$, whereas the equivalent from [8] relies on transitive concatenation of other translations and second it provides smaller target frameworks which might be crucial when applying translations for the reuse of sophisticated solvers.

Translation 1 ($com \Rightarrow (stage|stb|sem|prf)$)**.** *The transformation $Tr(A,R) = (A',R')$, with $A' = A \cup A^*$ and $R' = R \cup \{(a,b^*),(a^*,b) \mid (a,b) \in R\}$, is an embedding modular faithful translation for $com \Rightarrow (stage|stb|sem|prf)$.*

Proof. We take an AF $F = (A,R)$ as given and investigate $Tr(F)$. Observe that for any conflict-free set $E \subseteq A_F$ we have that E attacks a^* in $Tr(F)$ iff E attacks a in F and therefore E defends some argument a (and a^*) in $Tr(F)$ if and only if E defends a in F.

$E \in com(F) \Rightarrow E' = E \cup \{a^* \mid E \not\rightarrowtail^R a\} \in stb(Tr(F))$: First observe that A^* itself and thus by definition E' is conflict-free. Furthermore we have $\{a^* \mid a \in E\} \subseteq \{a^* \mid E \not\rightarrowtail^R a\} \subseteq E'$, as E is conflict-free by definition, thus $\{a, a^* \mid a \in E^+_F\} \subseteq E'^+_{Tr(F)}$. Now for any



(a) Translation 1 ($com \Rightarrow \sigma$).

(b) Translation 2 ($cf \Rightarrow naive$).

(c) Translation 3 ($cf \Rightarrow adm$, $naive \Rightarrow prf$).

(d) Translation 5 ($grd \Rightarrow (naive|stage)$).

(e) Translation 4 ($adm \Rightarrow com$, $naive \Rightarrow stage$, $prf \Rightarrow sem$).

(f) Translation 8 ($sem \Rightarrow prf$).

Figure 1. Four modular and two inefficient translations as applied to the AF from Example 1.

$a \in A_F \setminus E_F^+$ we have that, by completeness of E, a is attacked by some argument $b \notin E_F^+$. Hence $b^* \in E'$ and thus $E'^+ = A_{Tr(F)}$, i.e. $E' \in stb(Tr(F))$.

$E' \in prf(Tr(F)) \Rightarrow E = E' \cap A_F \in com(F)$: Due to the embedding property clearly E is conflict-free. For a contradiction let us assume that E is not admissible, i.e. there are $a \in E$, $b \in A_F$, $b \rightarrowtail^R a$ and E does not attack b. But then E' does not attack b^* in $Tr(F)$ and as b^* attacks a this contradicts E' being admissible. For completeness, again for a contradiction, let us assume there is an $a \in A_F \setminus E_F^+$ defended by E. As mentioned above then E (and E') also defend a in $Tr(F)$, contradicting maximality of E'. Hence $E \in com(F)$.

Furthermore since the difference between E and E' is to be found among the arguments A^* and due to maximality of E' we have $E' = E \cup \left\{a^* \mid E \not\rightarrowtail^R a\right\}$, marking proposed relations as bijections. Using the relations $stb(F) \subseteq sem(F) \subseteq prf(F)$ and equality of $stage$ and stb semantics where the latter is non-empty we obtain the assertion. □

Next we consider cf and $naive$ semantics. In the following translation, for each argument a we introduce a new argument $\bar{a}$ encoding that a is not in the extension.

Translation 2 ($cf \Rightarrow naive$)**.** *The transformation $Tr(A,R) = (A',R')$, with $A' = A \cup \bar{A}$ and $R' = R \cup \{(a,\bar{a}),(\bar{a},a) \mid a \in A\}$, is an embedding modular faithful translation for $cf \Rightarrow naive$.*

Proof. For $E \in cf(F)$ define $E' = E \cup \{\bar{a} \mid a \in A_F \setminus E\}$. Now E' is maximal conflict-free in $Tr(F)$ and thus $E' \in naive(Tr(F))$. On the other hand for $E' \in naive(Tr(F))$ we observe that for each argument $a \in A_F$ either $a \in E'$ or $\bar{a} \in E'$, thus the induced $E = E' \cap A_F$ is unique. Due to the embedding property it follows that $E \in cf(F)$. □

The next translation weakens the attack relation achieving symmetry such that admissibility and conflict-freeness coincide.

Translation 3 ($cf \Rightarrow adm$, $naive \Rightarrow prf$)**.** *The transformation $Tr(A,R) = (A,R')$, with $R' = R \cup \{(b,a) \mid (a,b) \in R\}$, is a covering modular exact translation for $cf \Rightarrow (cf|adm)$ and $naive \Rightarrow (naive|prf)$.*

Proof. We have that $Tr(F)$ is a symmetric framework. The results are immediate by the fact that the notion of admissibility and conflict-free coincide on such AFs. $\square$

Translation 4 ($adm \Rightarrow com$, $naive \Rightarrow stage$, $prf \Rightarrow sem$)**.** *The transformation $Tr(A,R) = (A',R')$, with $A' = A \cup \bar{A}$ and $R' = R \cup \{(a,\bar{a}),(\bar{a},a),(\bar{a},\bar{a}) \mid a \in A\}$, is an embedding modular exact translation for $adm \Rightarrow (com|adm)$, $naive \Rightarrow (naive|stage)$ and $prf \Rightarrow (sem|prf)$.*

Proof. Observe that by definition Tr equals Tr_1 from [8]. A detailed proof of $adm \Rightarrow (com|adm)$ and $prf \Rightarrow (sem|prf)$ is to be found there. We are left with showing that Tr is an exact translation for $naive \Rightarrow (naive|stage)$. In other words for any AF F we have (1) $naive(F) = naive(Tr(F))$ and (2) $naive(Tr(F)) = stage(Tr(F))$.
1) Since Tr is embedding and any $\bar{a} \in \bar{A}$ is self-conflicting we have that any $E \subseteq A_{Tr(F)}$ is conflict-free in F iff it is conflict-free in $Tr(F)$. Thus $naive(F) = naive(Tr(F))$.
2) Recall that any stage extension is also a naive extension. If $E \in naive(Tr(F))$ then $E^+_{Tr(F)} = E^+_F \cup \{\bar{a} \mid a \in E\}$. Consider $E' \in naive(Tr(F))$ such that $E^+_{Tr(F)} \subseteq E'^+_{Tr(F)}$ we receive $E \subseteq E'$ since any $\bar{a}$ with $a \in E$ is attacked only by a and $\bar{a}$. Thus with maximality of naive extensions $E' = E$ and therefore also $naive(Tr(F)) \subseteq stage(Tr(F))$. $\square$

We now study cases where no efficient translation exists and consider translations with arbitrary computational power. We start with an obvious translation for grounded semantics.

Translation 5 ($grd \Rightarrow (naive|stage)$)**.** *The transformation $Tr(A,R) = (A, R \cup \{(a,a) \mid a \in A \setminus grd(F)\})$ is a covering exact translation for $grd \Rightarrow (naive|stage)$.*

Notice that although Translation 5 is not efficient in the sense of Definition 4 it can be computed in polynomial time.

Due to Proposition 1 by leaving efficiency we as well leave modularity. Thus in the following we give three monotone translations covering a broad range of semantics.

Translation 6 ($\sigma \Rightarrow (stage|stb|sem|prf)$, $\sigma \Rightarrow com$)**.** *We define $Tr(A,R) = (A',R')$ as $A' = A \cup \{F',\ E_{F'} \mid F' \subseteq (A,R),\ E \in \sigma(F')\}$ and*

$$\begin{aligned}
R' = R \cup \{&(E_{F'},F'),(F',F'),(F',a) \mid a \in A_{F'}\} && (1)\\
\cup \{&(E_{F'},b) \mid b \in A_{F'} \setminus E\} && (2)\\
\cup \{&(E_{F'},E'_{F'}) \mid E_{F'} \neq E'_{F'}\} && (3)\\
\cup \{&(E_{F'},E_{F^<}),(E_{F'},F^<) \mid F^< \subsetneq F'\} && (4)
\end{aligned}$$

For semantics σ with $|\sigma(F)| \geq 1$ and $\sigma(F) \subseteq cf(F)$ [3] Tr is an embedding monotone faithful translation for $\sigma \Rightarrow (stage|stb|sem|prf)$ and a weakly faithful with remainder set $\emptyset$ translation for $\sigma \Rightarrow com$.

[3] As far as the semantics introduced in this work are concerned this excludes only stable semantics.

To achieve monotonicity we introduce arguments F' to represent subframeworks and arguments $E_{F'}$ to encode extensions of those subframeworks. The attacks in (1) and (2) ensure that a selected extension defends its arguments. The mutual attacks in (3) ensure that only one extension is selected while (4) ensures that only extensions of the whole framework are selected.

Proof. For AF F, extension $E \in \sigma(F)$ and thus $E \in cf(F)$ consider $E' = E \cup \{E_F\}$. E' is conflict-free since E attacks only (but all) those arguments from A_F not being member of E. Furthermore E_F attacks any E'_F with $E' \in \sigma(F), E' \neq E$, any $E_{F'}$ with $E \in \sigma(F'), F' \subsetneq F$ and any argument F' for $F' \subseteq F$. Hence $E' \in stb(Tr(F))$ and thus also $E' \in stage(Tr(F)) = sem(Tr(F))$ and $E' \in prf(Tr(F))$.

We now consider $E' \in prf(Tr(F))$. For any $E \in \sigma(F')$ with $F' \subsetneq F$ we have that $E_{F'}$ is not a member of E' since the only arguments defending $E_{F'}$ against the non-empty set $\{E_F \mid E \in \sigma(F)\}$ are members of this set and thus also attacking $E_{F'}$. Furthermore, by (3), at most one E_F is member of E'. We observe that there is no $D \in adm(Tr(F))$ such that $D \cap \{E_F \mid E \in \sigma(F)\} = \emptyset$, since all arguments from A are attacked by the argument F. We can thus pick the unique $E_F \in E' \cap \{E_F \mid E \in \sigma(F)\}$. But then E_F defends all arguments $a \in E$ and it follows immediately that $E' \cap A = E \in \sigma(F)$. □

Translation 7 ($grd \Rightarrow (sem|prf|com)$)**.** *We define the transformation* $Tr(A,R) = (A',R')$ *with* $A' = A \cup \{F' \mid F' \subseteq F\}$ *and*

$$R' = R \cup \{(F',F'),(F',a) \mid F' \subseteq (A,R), a \in A_{F'} \setminus grd(F')\} \tag{1}$$

$$\cup \{(a,F^<) \mid F^< \subsetneq F' \subseteq F, a \in A_{F'}\} \tag{2}$$

Tr is an embedding monotone exact translation for $grd \Rightarrow (sem|prf|com|grd)$*.*

Again for monotonicity we use arguments F' to represent subframeworks. The attacks in (1) ensure that only arguments from the grounded extension of the AF F' remain admissible in $Tr(F')$, while (2) ensures that proper subframeworks of F' are disabled.

Proof. The argument $F = (A,R)$ is attacked only by itself yet attacks any argument not being member of the grounded extension of F, thus disabling the arguments $A_F \setminus grd(F)$ for any admissible extension. If $grd(F) = \emptyset$ we clearly have $\emptyset$ as only extension for all the semantics of interest. Now consider the case where $grd(F) \neq \emptyset$. Then there is an argument $a \in A$ that is not attacked at all in F and therefore in all subframeworks of F. Hence a is in the grounded extension of all subframeworks containing a and thus in the grounded extension of $Tr(F)$ (as it is not attacked in $Tr(F)$). Now we can ignore all arguments $F' \subsetneq F$ since in $Tr(F)$ they are self-attacking and attacked by a. It follows that the grounded extension of F is also the grounded extension of $Tr(F)$. Further as all the other arguments are unacceptable the semantics of interest collapse. □

Translation 8 ($sem \Rightarrow prf$)**.** *We use* $\mathscr{P}(F')$ *to denote* $\mathscr{P}(F') = prf(F') \setminus sem(F')$*. The Transformation* $Tr(A,R) = (A',R')$ *with* $A' = A \cup \{P_{F'} \mid F' \subseteq (A,R), P \in \mathscr{P}(F')\}$ *and*

$$R' = R \cup \{(a,P_{F'}),(P_{F'},P_{F'}),(P_{F'},b) \mid a \in A_{F'} \setminus P_{F'}, b \in P_{F'}\} \tag{1}$$

$$\cup \{(a,P_{F^<}) \mid F^< \subsetneq F' \subseteq F, a \in A_{F'}\} \tag{2}$$

is an embedding monotone exact translation for $sem \Rightarrow prf$*.*

The idea behind above translation is that we eliminate preferred extensions which are not semi-stable by modifying the AF such that these extensions and their subsets are no longer admissible.

Proof. Observe that any conflict-free set in $Tr(F)$ $(F = (A,R))$ consists of arguments $a \in A$ only. Additional arguments of the form $P_{F'} \in \mathscr{P}(F')$ with $F' \subsetneq F$ are attacked by all $a \in A$, we can thus restrict ourselves to the set $S \subseteq A \cup \mathscr{P}(F)$. Now assume $E \in sem(A,R)$. Since $E \setminus P \neq \emptyset$ for any $P \in \mathscr{P}(F)$ we have that E attacks all $P_F \in Tr(F)$ and thus $E \in prf(Tr(F))$. On the other hand we might look at some $E \in prf(Tr(F))$ and assume for a contradiction that $E \notin sem(F)$. Then there has to be some $P \in \mathscr{P}(F)$ such that $E \subseteq P$. But now E is attacked by the argument $P_F \in Tr(F)$ and defended only by arguments $a \in A \setminus P$ and thus cannot be admissible. □

Now we can use the transitivity of translations to obtain $stage \Rightarrow prf$.

Corollary 1 ($stage \Rightarrow prf$)**.** *Considering Translation 8 as* Tr_8 *and Translation 2 in [8] as* $Tr_{stage,sem}$*.* $Tr = Tr_8 \circ Tr_{stage,sem}$ *is a covering monotone exact translation for* $stage \Rightarrow prf$*.*

4. Negative Results

In this section we study cases where exact or even faithful translations are impossible. The following theorem gives such impossibility results which rely on the incompatibility of very basic properties of semantics.

Theorem 1. *There is no translation which is*

1. *weakly faithful for* $(cf|naive|adm|stb|com|prf|sem|stage) \Rightarrow grd$*,*
2. *weakly exact for* $(cf|naive|grd|adm|com|prf|sem|stage) \Rightarrow stb$*,*
3. *weakly exact for* $(naive|grd|com|prf|sem|stage) \Rightarrow (adm|cf)$*,*
4. *weakly faithful for* $(naive|grd|adm|stb|com|prf|sem|stage) \Rightarrow cf$*,*
5. *weakly exact for* $(cf|adm|com) \Rightarrow (naive|stb|prf|sem|stage)$*.*

Proof. Any semantics of interest but *grd* can possess more than one extension implying (1). Any semantics of interest but *stb* can possess the empty set as an extension implying (2). The empty set on the other hand is always an admissible and thus conflict-free extension, while for the other semantics, depending on the concrete AF, the empty set may be an extension or not (3). For *cf* furthermore any subset of any extension is an extension which is not necessarily the case for the other semantics implying (4). Only for *cf*, *adm* and *com* semantics we have that extensions may form proper subsets implying (5). □

Next we consider the cases which are not already covered by Theorem 1.

Theorem 2. *There is no weakly exact translation for* $(naive|prf|sem|stage) \Rightarrow com$*.*

Proof. We assume for a contradiction that such a translation Tr exists. Now observe that for $\sigma \in \{naive, prf, sem, stage\}$ there are AFs such that $\emptyset \in \sigma(F)$. Thus $\emptyset$ is not a member of the remainder sets of Tr.

Now take into account the AF $F = (A,R)$ with $A = \{a,b\}$ and $R = \{(a,b),(b,a)\}$. Then $\sigma(F) = \{\{a\},\{b\}\}$. The grounded extension is the least complete extension, thus with $\{a\},\{b\} \in com(Tr(F))$ we need $\emptyset = grd(Tr(F))$ and thus $\emptyset \in com(Tr(F))$. □

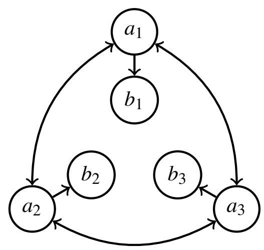


Figure 2. Argumentation framework serving as a counter example for Theorem 3.

Theorem 3. *There is no weakly faithful translation* $\{stage, stb, sem, prf, com, adm\} \Rightarrow \{cf, naive\}$.

Proof. For a contradiction we assume that any such translation $Tr : \sigma \Rightarrow \sigma'$ exists. Consider the AF $F = (A, R)$ as shown in Figure 2. Now observe that for $E_1 = \{a_1, b_2, b_3\}, E_2 = \{b_1, a_2, b_3\}, E_3 = \{b_1, b_2, a_3\}, B = \{b_1, b_2, b_3\}$ we have that E_1, E_2, E_3 are σ-extensions while $B \notin \sigma(F)$. So for any E_i there has to be some $E'_i \in \sigma'(Tr(F))$ such that $E_i \subseteq E'_i$. Thus immediately $B \in cf(Tr(F))$, since pairwise conflict-freeness of the b_i is granted by E'_1, E'_2 and E'_3. For any conflict-free set B in any AF F' there has to be some extension $E \in naive(F')$ such that $B \subseteq E$, yielding a contradiction also for naive semantics. □

In the preceding section we presented various translations of different framework similarity levels. As far as modular translations are concerned impossibility of weakly exact translations $stb \Rightarrow (prf|com|adm)$, $stage \Rightarrow sem$ and weakly faithful translations $sem \Rightarrow stage$ can be shown. Especially the proof of the latest result turns out to be complicated and due to focus on inefficient translations has to be delayed to subsequent works. However in the following we will give impossibility results for embedding respectively monotone translations thus showing that the translations given in Section 3 are optimal wrt. the translational properties of interest.

Theorem 4 ($grd \Rightarrow (naive|stage)$)**.** *There is no translation which is*

1. *embedding or monotone weakly faithful for* $grd \Rightarrow naive$*.*
2. *embedding or monotone weakly exact for* $grd \Rightarrow stage$*.*

Proof. For the semantics of interest we observe that for embedding or monotone translations Tr with $a \in A_F$ immediately also $a \in A_{Tr(F)}$. Furthermore due to expandability we have $(a,a) \in A_{Tr(F)} \iff (a,a) \in A_F$. We refer to these observations by the term inheritance for the realm of this proof.

Take into account the AFs $F = (A, R)$ and $F' = (A, R')$ with $A = \{a, b, c\}$, $R = \{(b,c), (c,b)\}$ and $R' = R \cup \{(a,b)\}$. Now $grd(F) = \{a\}$ and $grd(F') = \{a, c\}$. For a contradiction we assume existence of a translation $Tr : grd \Rightarrow \sigma$ of the desired kind. For $\sigma = naive$ due to inheritance we deduce that $(c,c) \notin Tr(F)_R$. Hence there exists an extension $E \in naive(Tr(F))$ with $c \in E$ the latter implying that E cannot be a remainder set, a contradiction. For $\sigma = stage$ we observe that due to inheritance and exactness there has to be some conflict between a and c in $Tr(F)$, thus Tr cannot be embedding. If Tr is monotone then from $F \subseteq F'$ we conclude that the conflict between a and c also occurs in $Tr(F')$, a contradiction to $\{a, c\} \in naive(Tr(F))$. □

Table 2. Final results for (weakly) faithful / exact translations.

	cf	*naive*	*grd*	*adm*	*stb*	*com*	*prf*	*sem*	*stage*
cf	✓	✓/ -	–	✓	✓/ -	✓	✓/ -	✓/ -	✓/ -
naive	–	✓	–	✓/ -	✓/ -	✓/ -	✓	✓	✓
grd	–	✓	✓	✓/ -	✓/ -	✓	✓	✓	✓
adm	–	–	–	✓	✓/ -	✓	✓/ -	✓/ -	✓/ -
stb	–	–	–	✓	✓	✓	✓	✓	✓
com	–	–	–	✓/ -	✓/ -	✓	✓/ -	✓/ -	✓/ -
prf	–	–	–	✓/ -	✓/ -	✓/ -	✓	✓	✓/ -
sem	–	–	–	✓/ -	✓/ -	✓/ -	✓	✓	✓/ -
stage	–	–	–	✓/ -	✓/ -	✓/ -	✓	✓	✓

Theorem 5. *There is no translation for* $(cf|naive) \Rightarrow (stb|sem|prf|com|adm)$ *which is embedding and weakly exact.*

Proof. For a contradiction we assume that such a translation Tr happens to exist. We take into account the AF $F = (\{a,b\},\{(a,b)\})$. We have $cf(F) = \{\{a\},\{b\},\emptyset\}$ and $naive(F) = \{\{a\},\{b\}\}$. Since b is attacked by a in F and with the premise of embedding in mind for any admissible set E with $b \in E$ we need E to attack a. So either b attacks a and the translation is not embedding or some from b different argument $c \in E$ attacks a and the translation is not exact. With the observation that stable, semi-stable, preferred and complete semantics are all based on admissibility we finish this proof. □

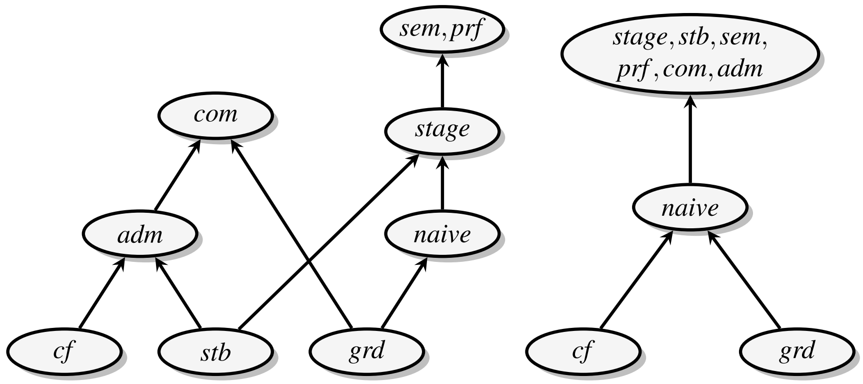
Remark. The AF $F = (A,R)$ with $A = \{a,b,c\}$ and $R = \{(a,b),(b,c),(c,c)\}$ is used in [8] to show impossibility of embedding weakly exact translations $stage \Rightarrow sem$. Immediately the same holds for embedding weakly exact translations $stage \Rightarrow prf$.

5. Conclusion

We studied expressiveness of argumentation semantics using translations, and in contrast to previous work on intertranslatability we did not restrict ourselves to efficiently computable transformations. Our results together with those from [8] are stated in Table 2.

Our results strengthen preliminary knowledge in two ways. First it appears that certain translations remain impossible regardless of the computational effort one is willing to pay. Second, in cases where no efficient translation exists, we showed that translations satisfying in fact still nice properties are possible if we accept high computational costs.

Figure 3 visualises the hierarchies of expressiveness of the chosen argumentation semantics for exact translations (a) and faithful translations (b). A solid path from a semantic σ to a semantics σ' expresses that there is a translation for $\sigma \Rightarrow \sigma'$. Furthermore, if for two semantics σ, σ' there is no path from σ to σ' then it is proven that there is no such faithful translation for $\sigma \Rightarrow \sigma'$. Let us highlight some differences to the picture drawn for efficient translations (see [8]). When neglecting computational costs we have exact translations for $grd \Rightarrow (com|stage|prf|sem)$ while for efficient translations the first is impossible and the others are still open. Concerning (weakly) faithful translations we have that $stage, stb, sem, prf, com, adm$ can be translated to each other while when considering efficient (weakly) faithful translations these semantics form at least three levels of intertranslatability.



(a) (weakly) exact

(b) (weakly) faithful

Figure 3. Results for inefficient intertranslatability.

Finally, when motivating translations by expressiveness and meta-level discussion (and not by computational issues) argument labelings (see e.g. [5]) distinguishing different kind of arguments which are not accepted by an extension are of additional interest. As preserving the labels of an argument when translating AFs needs additional effort it is not clear whether positive results carry over to the labeling setting, while the obtained negative results immediately hold for labelings. We leave this subject as future work.

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