Modified Marching Octahedra for Optimal Regular Meshes



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Figure 1: The body-centred cubic mesh

1 Mesh Construction

Theußl et al. [Theußl et al. 2001] showed that volumetric data sampled on a body-centred cubic (BCC) lattice is nearly 30% more efficient than data sampled on a cubic lattice, and produced volume renderings using splatting. We extend this work to generate isosurfaces based on the BCC lattice, and also on the hexagonal-close packed (HCP) grid. This sketch presents a modified version of marching octahedra that simplifies the BCC mesh to an octahedral mesh to reduce the number of triangles generated for the isosurface.

The marching cubes technique [Lorenson and Cline 1987; Wyvill et al. 1986] uses a cubic cells: the Delaunay complex of the samples. The Delaunay complex for HCP uses tetrahedra (*tets*) and octahedra (*octs*). The Delaunay complex for BCC (Fig. 1(a)) has two superimposed cubic meshes, shown in blue (*primary lattice*) and green (*secondary lattice*), with diagonals between the two meshes, shown in red.

2 (Modified) Marching Octahedra

Tetrahedral cells can be dealt with by marching tets [Bloomenthal 1988]. We define six cases for marching octs using the same methodology as for marching cubes and marching tets. Only one case is ambiguous (Fig. 2). The faces of an oct are triangular, not square, so the cracks of marching cubes [Dürst 1988] do not occur, and we can choose either case without causing cracks in the surface. Although tets use fewer triangles than cubes for isosurfaces, there are many more tets in the BCC grid than cubes in the cubic grid. We address this by grouping tets into octs.

In the BCC lattice, octs composed of groups of four tets share each edge of the secondary lattice, as shown in



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Figure 2: Case 3 of Modified Marching Octahedra

Fig. 1(b). We refer to the shared edge as the *spine* of the oct. If the two vertices "above" the isosurface are along the spine of the oct, marching tets renders the surfaces shown in Fig. 2(b). If not, marching tets renders the surfaces shown in Fig. 2(a). We use the location of the spine to choose case 3A or 3B, guaranteeing that the surface generated is topologically equivalent to that generated by marching tets. By substituting octs for tets, we attempt to reduce the number of triangles rendered for an isosurface on a BCC lattice.

3 Expected Triangle Counts

We estimate the number of triangles generated by assuming that $O(N^{2/3})$ cells intersect each isosurface, ignoring constants. We adjust N for BCC and HCP to reflect the 29.3% saving in samples noted above, and multiply by an estimate of the number of triangles generated per cell - 2.85 for cubes, 1.25 for tets, 4.07 for octs (modified). This gives the following estimates for similar quality sampling: $2.85N^{2/3}$ (cubes), $3.42N^{2/3}$ (HCP), $3.75N^{2/3}$ (BCC), $4.23N^{2/3}$ (BCC with octs). We are evaluating triangle counts experimentally: preliminary results are similar to these estimates.

References

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